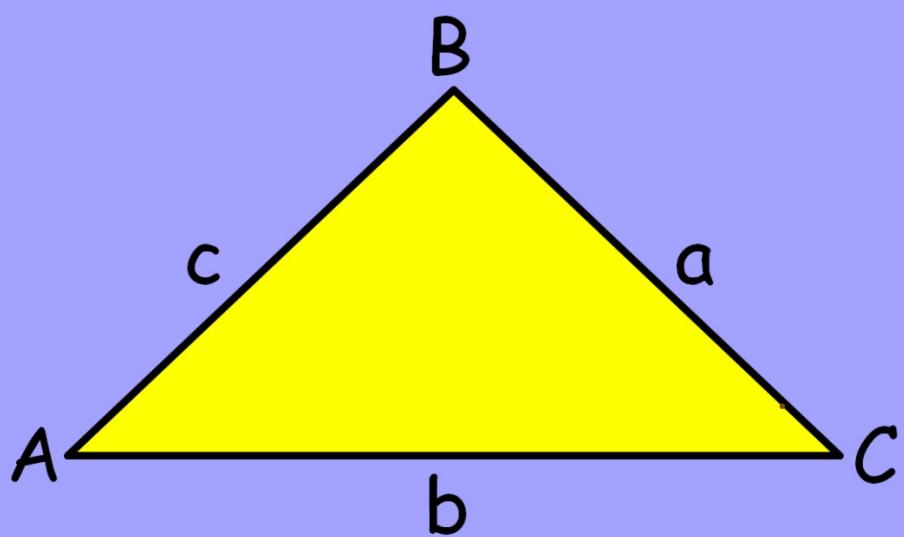


Law of Sines and Cosines

Objective:

To use trigonometry to solve oblique (non-right) triangles.

Standard Notation for a Triangle



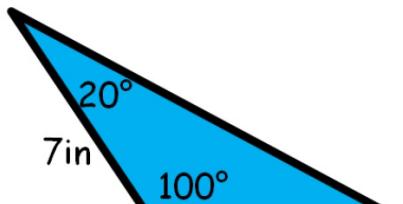
A, B, and C refer to the angles of the triangle.
a, b, and c refer to the side lengths of the triangle.

To be able to solve a triangle* you need to know...

- 1) One side length.
- 2) Any other 2 pieces of information (for example 1 additional side length and 1 angle measure.)

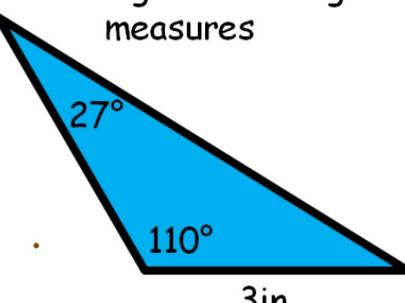
"to solve a triangle" means that you find all three side lengths and all three angle measures.

1 side length and 2 angle measures



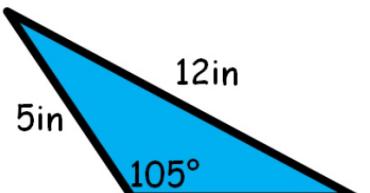
ASA

1 side length and 2 angle measures



AAS

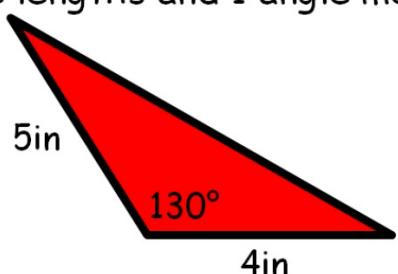
2 side lengths and 1 angle measure



SSA

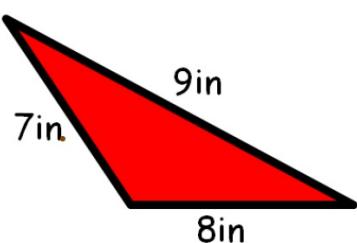
LAW OF SINES

2 side lengths and 1 angle measure



SAS

3 side lengths



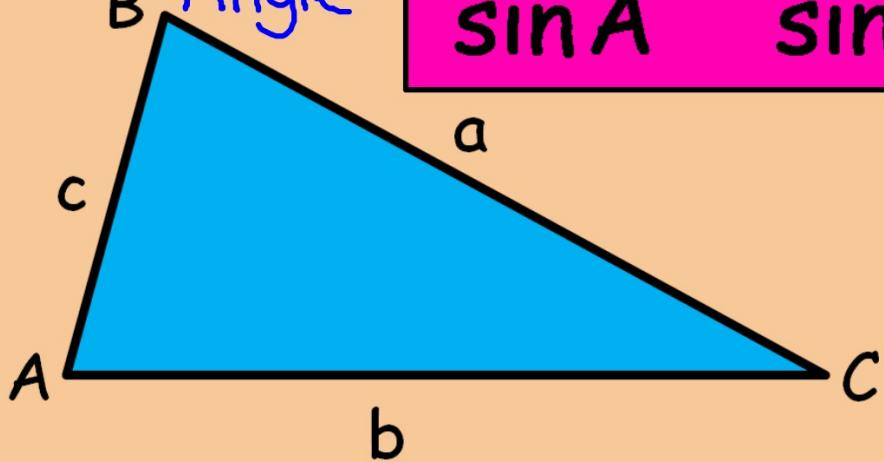
SSS

LAW OF COSINES

Law of Sines -

If $\triangle ABC$ is a triangle with sides a , b , and c , then

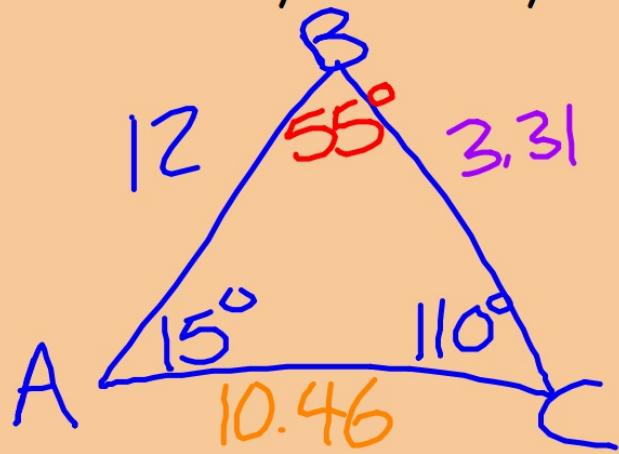
Side
Angle



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Example 1:

$$A = 15^\circ, C = 110^\circ, c = 12 \text{ in}$$



$$\frac{12}{\sin(110)} = \frac{a}{\sin(15)}$$

$$\frac{a \sin(110)}{\sin(110)} = \frac{12 \sin(15)}{\sin(110)}$$

$$\frac{12}{\sin(110)} = \frac{b}{\sin(55)}$$

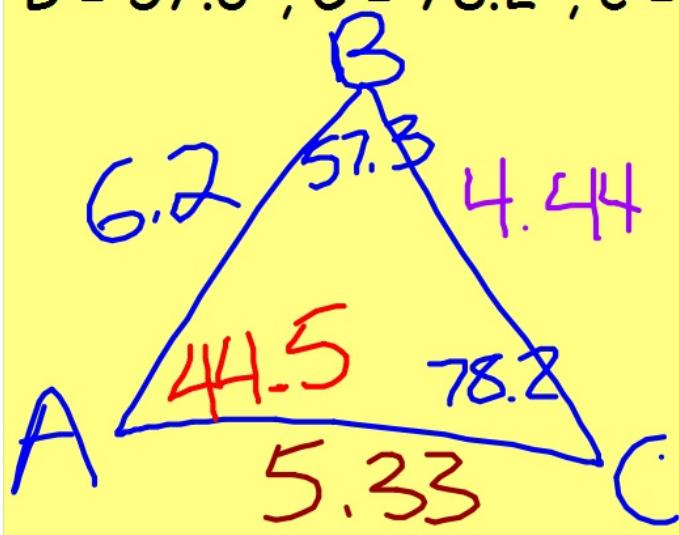
Does it
make
sense?!

$B = 55^\circ$
$a = 3.31$
$b = 10.46$

$$b = 10.46$$

Example 2 -

$$B = 57.3^\circ, C = 78.2^\circ, c = 6.2\text{cm}$$



$$\frac{6.2}{\sin(78.2)} = \frac{a}{\sin(44.5)}$$
$$a = \frac{6.2 \sin(44.5)}{\sin(78.2)}$$
$$a = 4.44$$

$$\frac{6.2}{\sin(78.2)} = \frac{b}{\sin(57.3)}$$

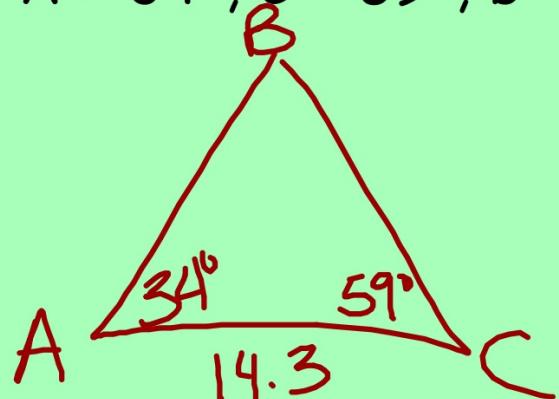
Does it
make
sense?!

$A = 44.5^\circ$
$a = 4.44$
$b = 5.33$

$$b = 5.33$$

Example 3 -

$$A = 34^\circ, C = 59^\circ, b = 14.3\text{cm}$$



B =	_____
a =	_____
c =	_____

Example 4 -

$$A = 28^\circ, B = 62^\circ, b = 19\text{cm}$$

$C =$ _____
$a =$ _____
$c =$ _____

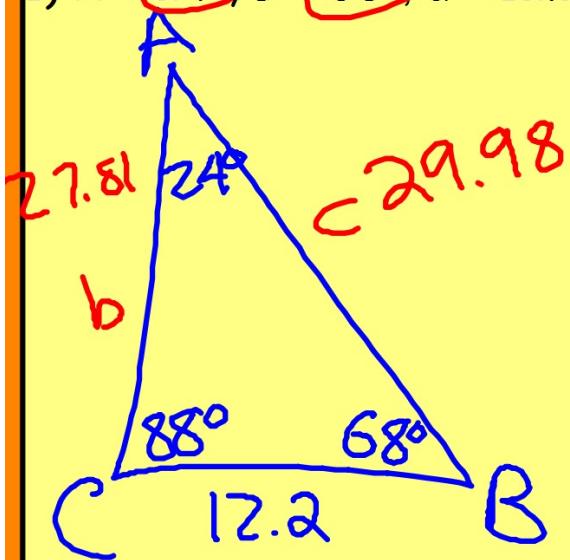
1/12 HW Answers

- ① $A = 36^\circ$ $a = 14.72$ $c = 23.53$
- ② $C = 46$ $a = 6.53$ $b = 6.25$
- ③ $B = 43^\circ$ $b = 6.66$ $c = 9.67$
- ④ $C = 75$ $a = 24.89$ $b = 30.48$
- ⑤ $A = 49^\circ$ $a = 41.80$ $c = 53.96$
- ⑥ $A = 26^\circ$ $a = 10.44$ $b = 21.03$

End of 1/12 Notes

Warm Up - solve for the missing sides and/or angles

1) $A = 24^\circ$, $B = 68^\circ$, $a = 12.2\text{ft}$



$$c = \frac{88}{\sin(24)}$$

$$b = \frac{27.81}{\sin(24)}$$

$$c = \underline{\underline{29.98}}$$

2) $B = 104^\circ$, $C = 33^\circ$, $a = 18.1$

$$\frac{12.2}{\sin(24)} = \frac{b}{\sin(68)}$$

$$\frac{12.2 \sin 68}{\sin(24)} = \frac{b \sin(68)}{\sin(24)}$$

$$27.81 = b$$

$$A = \underline{\underline{\hspace{2cm}}}$$

$$b = \underline{\underline{\hspace{2cm}}}$$

$$c = \underline{\underline{\hspace{2cm}}}$$

The Ambiguous Case: SSA

- No Solution (No such triangle exists)
- One Solution (One triangle exists)
- Two Solutions (Two triangles satisfy the conditions)

REMEMBER:

If it spells out SSA, then it is
a pain in the...

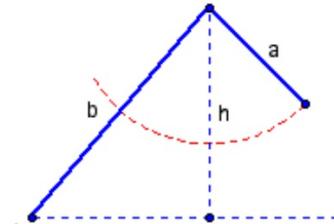
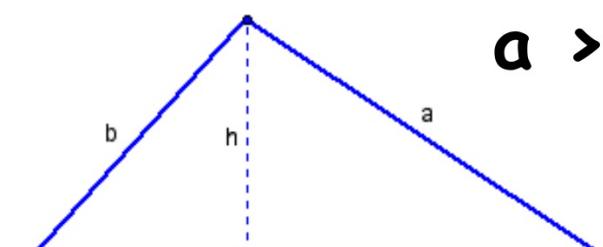
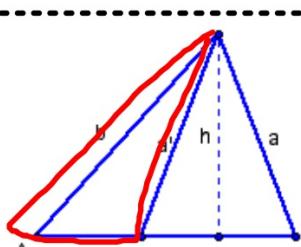
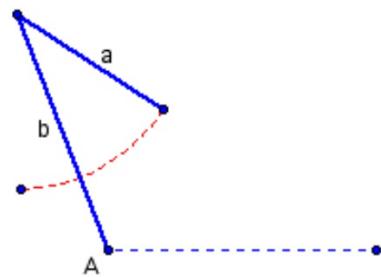
	Diagram	Triangles Possible
Given angle is acute	<p>If $a < b$ but $a < h$</p> 	None
	<p>$a > b$</p> 	One
	<p>$h < a < b$</p> 	Two

Diagram Triangles Possible

Given
angle is
obtuse

$\angle A$ is
obtuse



$$a \leq b$$

$$a > b$$

None

One

How many Solutions?

0 or 2

Side across from given angle is smaller

1

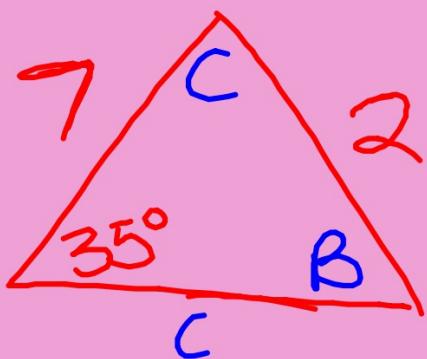
Side across from given angle is bigger

Example 1:

SSA

$$\underline{A} = 35^\circ, b = 7 \text{ in}, \underline{a} = 2 \text{ in}$$

0 or 2



$$\frac{2}{\sin(35)} = \frac{7}{\sin(B)}$$

$$\frac{7 \sin(35)}{2} = \frac{2 \sin(B)}{2}$$

$$\frac{7 \sin(35)}{2} = \frac{\sin(B)}{2}$$
$$\sin^{-1}\left(\frac{7 \sin(35)}{2}\right) = B$$

No triangle

B =	Exists
C =	_____
c =	_____

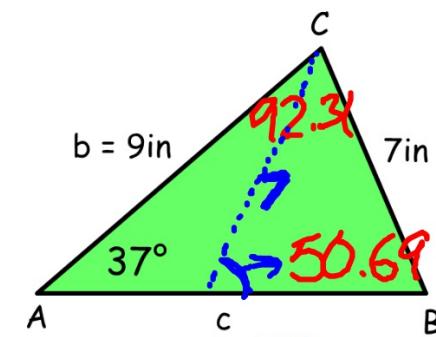
Example 2: _____

$A = 37^\circ$, $b = 9\text{in}$, $a = 7\text{in}$

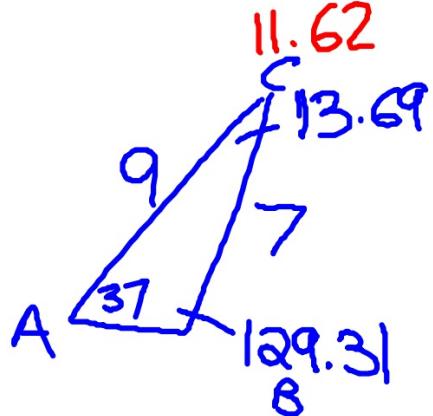
$B =$ _____

$C =$ _____

$c =$ _____



First
Triangle



$$\frac{7}{\sin(37)} = \frac{c}{\sin(13.69)}$$

$$B = \underline{129.31}$$

$$C = \underline{13.69}$$

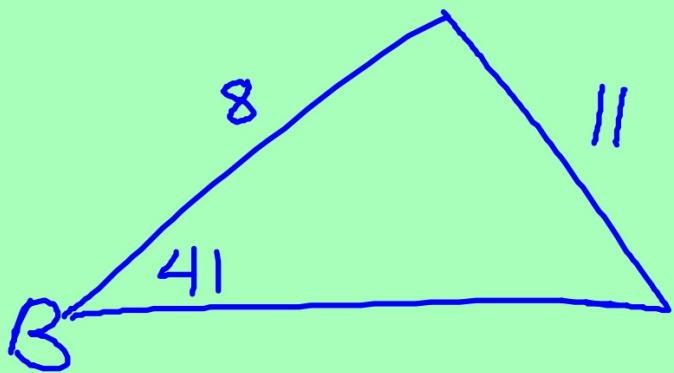
$$c = \underline{2.75}$$

This is NOT a unique triangle. This is why SSA does NOT always work as a congruence theorem in Geometry!

Example 3: SSA

B = 41° , b = 11cm, c = 8cm

①



$$\begin{aligned} A &= \underline{110.50^\circ} \\ C &= \underline{28.50^\circ} \\ a &= \underline{15.70^\circ} \end{aligned}$$

Example 4: _____

$A = 102^\circ$, $b = 5\text{in}$, $a = 8\text{in}$

B = _____
C = _____
c = _____

Example 5: _____

$A = 120^\circ$, $a = 9\text{in}$, $c = 4\text{in}$

B = _____

C = _____

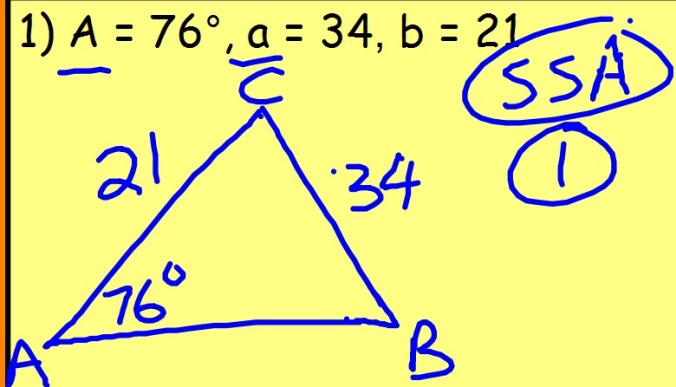
b = _____

Homework

1. $B = 130^\circ$ $a = 10$ $b = 8$
2. $A = 20^\circ$ $a = 10$ $c = 11$
3. $C = 95^\circ$ $a = 8$ $c = 9$
4. $A = 70^\circ$ $B = 60^\circ$ $c = 25$
5. $C = 16^\circ$ $b = 92$ $c = 32$
6. $A = 10^\circ$ $C = 130^\circ$ $b = 5$

Warm Up - solve for the missing sides and/or angles

1) $A = 76^\circ, a = 34, b = 21$



2) $A = 53^\circ, a = 15, c = 18$

SSA
0 or 2

$$\frac{34}{\sin 76^\circ} = \frac{21}{\sin(B)}$$

$$B = \underline{\underline{36.82^\circ}}$$

$$C = \underline{\underline{67.18^\circ}}$$

$$\therefore c = \underline{\underline{32.30}}$$

$$B = \underline{\underline{53.59^\circ}}$$

$$C = \underline{\underline{73.41^\circ}}$$

$$b = \underline{\underline{15.12}}$$

Other set of solutions?

Other set of solutions?

HOMEWORK: Solutions

page 804 #29-34

1) SSA No Solution

2) SSA Two Solutions

$$B = 137.90^\circ \quad C = 22.10^\circ \quad b = 19.60$$

$$B = 2.1^\circ \quad C = 157.9^\circ \quad b = 1.07$$

3) SSA One Solution

$$A = 62.31^\circ \quad B = 22.69^\circ \quad b = 3.48$$

4) ASA One Solution!

$$C = 50^\circ \quad a = 30.67 \quad c = 28.26$$

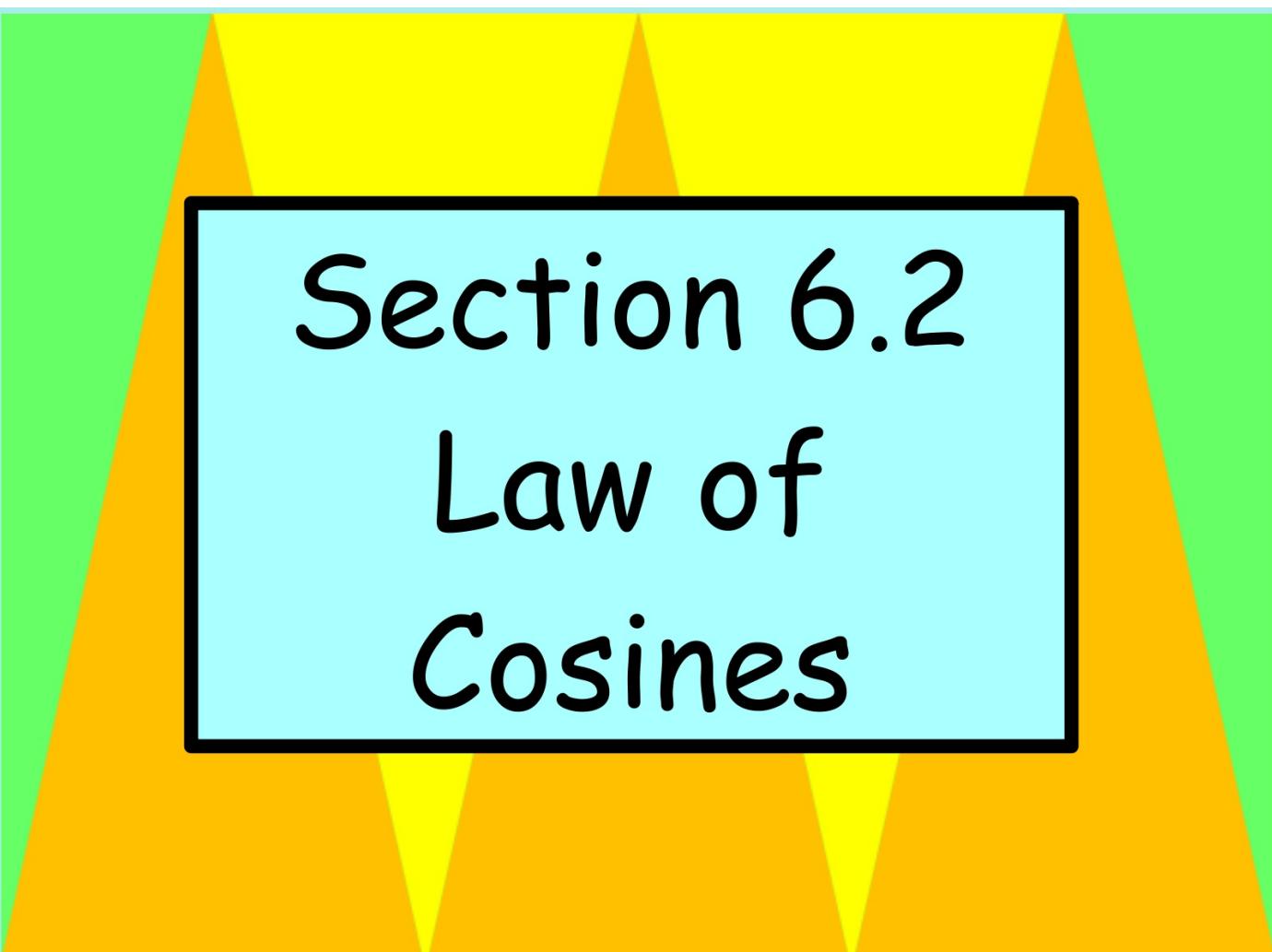
5) SSA Two Solutions!

$$A = 111.58^\circ \quad B = 52.42^\circ \quad a = 107.96$$

$$A = 36.42^\circ \quad B = 127.58^\circ \quad a = 68.93$$

6) ASA One Solution

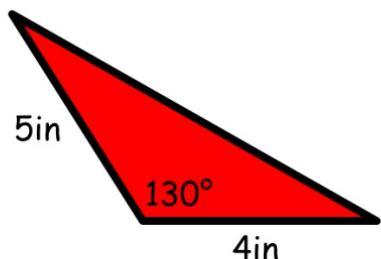
$$B = 40^\circ \quad a = 1.35 \quad c = 5.96$$



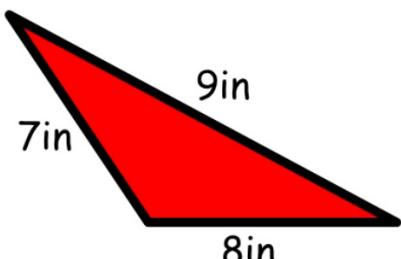
Section 6.2

Law of Cosines

- Used to solve triangles that are not right triangles
- Used when there is no corresponding angle and side given.



SAS

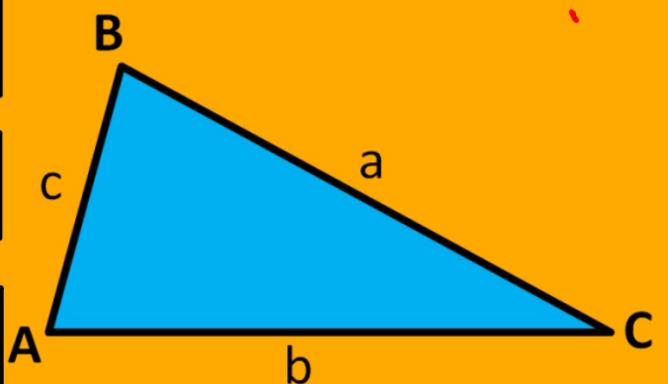


SSS

LAW OF COSINES:

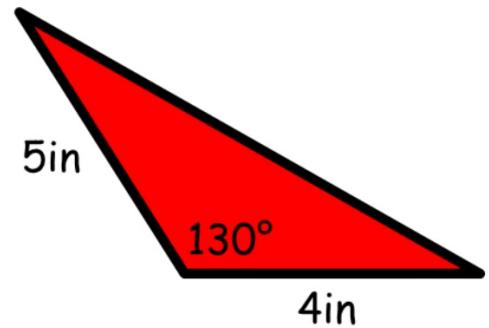
$$a^2 = b^2 + c^2 - (2bc)\cos A$$

∴



Law of Cosines

CASE 1: SAS

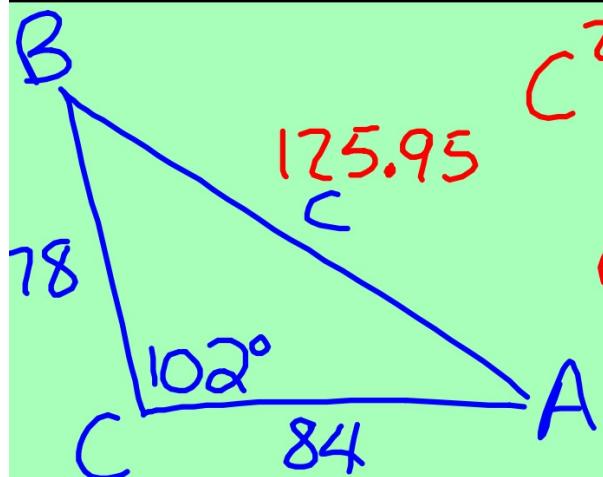


- 1.) Fill givens into the Law of Cosines option that corresponds to the given angle. Find the missing side length.
- 2.) Use Law of Sines to find the smallest missing angle measure.
- 3.) Subtract from 180 to find the last missing angle measure.



Example 1: SAS

$$c = 102^\circ, b = 84, a = 78$$



$$c^2 = 78^2 + 84^2 - 2 \cdot 78 \cdot 84 \cdot \cos(102^\circ)$$

$$c^2 = 15864.47$$

$$c = 125.95$$

$$\frac{125.95}{\sin(102^\circ)} = \frac{78}{\sin(A)}$$

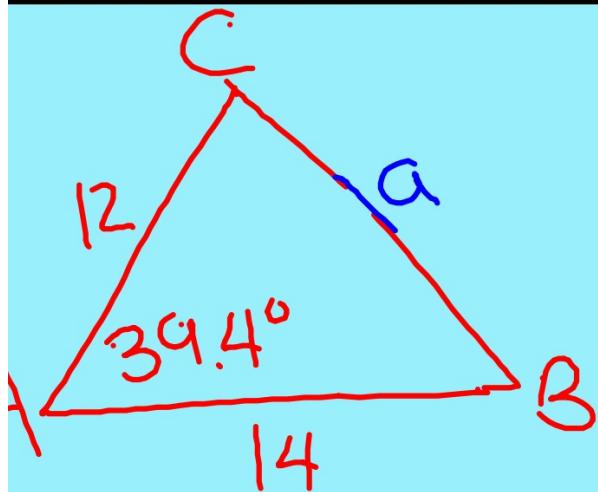
$$A = \underline{\underline{37.28^\circ}}$$

$$B = \underline{\underline{40.72^\circ}}$$

$$C = \underline{\underline{125.95}}$$

Example 2: SAS

A = 39.4° , c = 14, b = 12

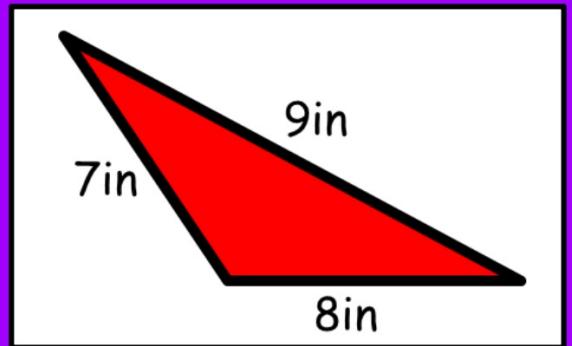


$$a^2 = 12^2 + 14^2 - 2 \cdot 12 \cdot 14 \cdot \cos(39.4)$$
$$a = 8.96$$

B =	<u>58.22°</u>
C =	<u>82.38°</u>
a =	<u>8.96</u>

Law of Cosines

CASE 2: SSS



- 1.) Fill into the three Law of Cosines Options to get the smallest angle measure.

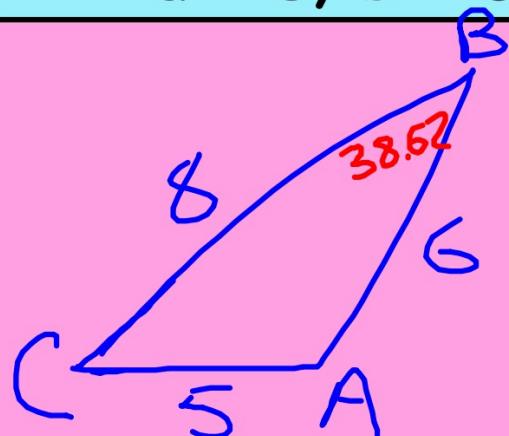
- 2.) Use Law of Sines to find the next smallest missing angle measure.

- 3.) Subtract from 180 to find the largest angle measure.



Example 3: SSS

$$a = 8, b = 5, c = 6$$



$$5^2 = 6^2 + 8^2 - 2 \cdot 6 \cdot 8 \cdot \cos(B)$$

$$25 = 36 + 64 - 96 \cos(B)$$

$$25 = 100 - 96 \cos(B)$$

$$-100 \quad -100$$

$$\frac{75}{96} = \cos(B)$$

$$\frac{-75}{-96} = \frac{-96 \cos(B)}{-96}$$

$$\cos^{-1}\left(\frac{75}{96}\right) = B$$

$$A = \underline{\underline{92.88^\circ}}$$

$$B = \underline{\underline{38.62^\circ}}$$

$$C = \underline{\underline{48.50^\circ}}$$

Example 4: a = 19, b = 24.3, c = 21.8

$$\begin{aligned}A &= \underline{\underline{48.30^\circ}} \\B &= \underline{\underline{72.76^\circ}} \\C &= \underline{\underline{58.94^\circ}}\end{aligned}$$

HW #34

- 1) $B = 20^\circ$ $a = 120$ $c = 100$
- 2) $C = 95^\circ$ $a = 10$ $b = 12$
- 3) $a = 25$ $b = 11$ $c = 24$
- 4) $a = 2$ $b = 4$ $c = 5$
- 5) $a = 5$ $C = 24^\circ$ $b = 8$
- 6) $a = 32$ $b = 39$ $c = 16$

HOMEWORK: Solutions

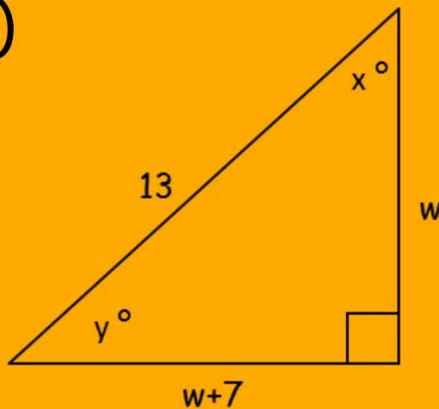
page 810 #5-8, 15-17

- 1) SAS $A = 107.4^\circ$ $C = 52.7^\circ$ $b = 43.0$**
- 2) SAS $A = 37.7^\circ$ $B = 47.3^\circ$ $c = 16.3$**
- 3) SSS $A = 82.2^\circ$ $B = 25.8^\circ$ $C = 72.0^\circ$**
- 4) SSS $A = 22.3^\circ$ $B = 49.5^\circ$ $C = 108.2^\circ$**
- 5.) SAS $A = 35.2^\circ$ $B = 112.8^\circ$ $a = 4.60$**
- 6.) SSS $A = 52.9^\circ$ $B = 103.6^\circ$ $C = 23.5^\circ$**

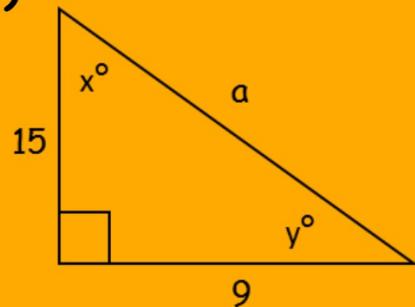
Trig and Special Right Triangle Review

-Solve for all missing sides and angles-

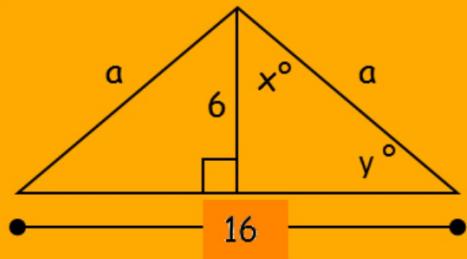
1)



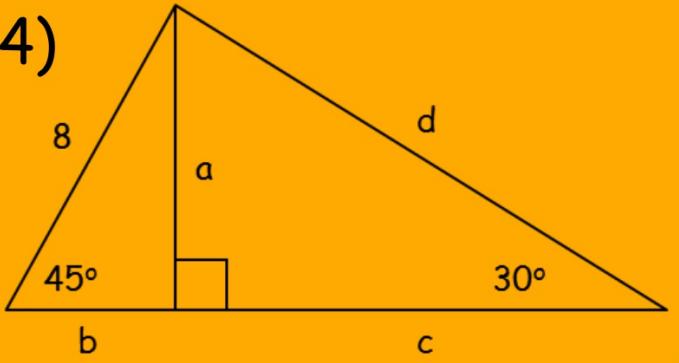
2)



3)



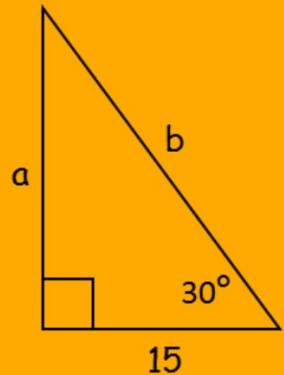
4)



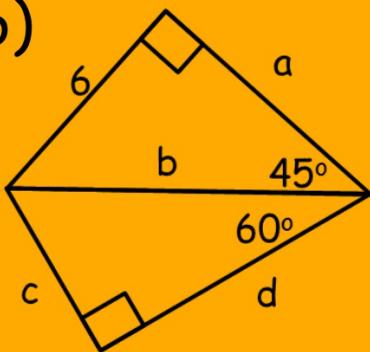
Trig and Special Right Triangle Review

-Solve for all missing sides and angles-

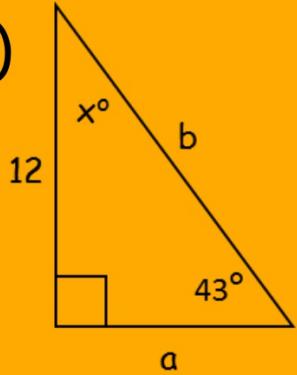
5)



6)



7)



8)

